Integrals

Assume α, β and t_0 are complex numbers. For definite integrals, assume also that $\Re(\alpha) > 0$. The square root ambiguity should be resolved so that $\Re(\sqrt{\alpha}) > 0$ as well.

$$\int t^n e^{\alpha t}\,dt = \frac{1}{\alpha} t^n e^{\alpha t} - \frac{n}{\alpha}\int t^{n-1} e^{\alpha t}\,dt$$

$$\int t^n \cos(\alpha t)\,dt = \frac{1}{\alpha} t^n \sin(\alpha t) - \frac{n}{\alpha}\int t^{n-1}\sin(\alpha t)\,dt$$

$$\int t^n \sin(\alpha t)\,dt = -\frac{1}{\alpha} t^n \cos(\alpha t) + \frac{n}{\alpha}\int t^{n-1}\cos(\alpha t)\,dt$$

$$\int \cos^n(\alpha t)\,dt = \frac{1}{\alpha n}\cos^{n-1}(\alpha t)\sin(\alpha t) + \frac{n-1}{n}\int \cos^{n-2}(\alpha t)\,dt$$

$$\int \sin^n(\alpha t)\,dt = -\frac{1}{\alpha n}\sin^{n-1}(\alpha t)\cos(\alpha t) + \frac{n-1}{n}\int \sin^{n-2}(\alpha t)\,dt$$

$$\int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha} \qquad \int_0^\infty t^n e^{-\alpha t}\,dt = \frac{n!}{\alpha^{n+1}}$$

$$\int_0^\infty t e^{-\alpha t^2}\,dt = \frac{1}{2\alpha} \qquad \int_{-\infty}^\infty t e^{-\alpha t^2}\,dt = 0$$

$$\int_0^\infty e^{-\alpha t^2}\,dt = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad \int_{-\infty}^\infty e^{-\alpha t^2}\,dt = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\infty,0}^\infty t^n e^{-\alpha t^2}\,dt = \frac{n-1}{2\alpha}\int_{-\infty,0}^\infty t^{n-2} e^{-\alpha t^2}\,dt$$

$$\int_{-\infty}^\infty \exp(-\alpha(t-t_0)^2 + \beta t)\,dt = \sqrt{\frac{\pi}{\alpha}}\exp\left(\beta t_0 + \frac{\beta^2}{4\alpha}\right)$$

$$\int_{-\infty}^\infty \frac{\sin(t)}{t}\,dt = \pi$$

$$f(x) = \int_{-\infty}^\infty \hat{f}(k) e^{ikx}\,dk \iff \hat{f}(k) = \frac{1}{2\pi}\int_{-\infty}^\infty f(x) e^{-ikx}\,dx$$